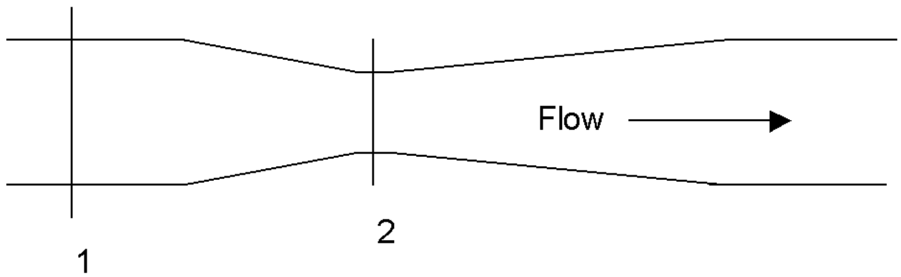
Derivation of Venturi Tube Flow Calculations from Chapter 15.

Page 442, Equations 15-3, 15-4 and 15-5 are printed incorrectly in the textbook. The proper denominator should be: $(A_1/A_2)^2-1$

This equation is derived directly from Bernoulli's Equation. The confusion comes from this denominator changing depending on if you are solving velocity of the main pipe or the velocity in the venturi. I will derive it both ways, but in either case, Position 1 will be the pipe before the venturi, and Position 2 will be at the venturi.



$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$$Q = A_1 v_1 = A_2 v_2 \quad where \quad v_2 = v_1\left(\frac{A_1}{A_2}\right) \quad and \quad v_1 = v_2\left(\frac{A_2}{A_1}\right)$$

Solving for v_1 : first, combine like terms:

$$\left(\frac{P_1 - P_2}{\gamma}\right) + (z_1 - z_2) - h_L = \left(\frac{v_2^2 - v_1^2}{2g}\right)$$

And multiplying both sides by 2g:

$$2g\left[\left(\frac{P_1 - P_2}{\gamma}\right) + (z_1 - z_2) - h_L\right] = v_2^2 - v_1^2$$

Now substitute for v_2:

$$2g\left[\left(\frac{P_1 - P_2}{\gamma}\right) + (z_1 - z_2) - h_L\right] = \left(v_1\left(\frac{A_1}{A_2}\right)\right)^2 - v_1^2 = v_1^2\left(\frac{A_1}{A_2}\right)^2 - v_1^2$$

Factor out v_1^2 from the right side of the equation:

$$2g\left[\left(\frac{P_1-P_2}{\gamma}\right)+(z_1-z_2)-h_L\right]=v_1^2\left[\left(\frac{A_1}{A_2}\right)^2-1\right]$$

And divide by the $(A_1/A_2)^2$-1 factor and then take the square root of both sides:

$$\sqrt{\frac{2g\left[\left(\frac{P_1-P_2}{\gamma}\right)+(z_1-z_2)-h_L\right]}{\left[\left(\frac{A_1}{A_2}\right)^2-1\right]}}=v_1$$

Note that the denominator is different than what is printed in the textbook.

If you solve for the velocity in the venturi, (v_2) you substitute for v_1 and get:

$$2g\left[\left(\frac{P_1-P_2}{\gamma}\right)+(z_1-z_2)-h_L\right]=v_2^2-\left(v_2\left(\frac{A_2}{A_1}\right)\right)^2=v_2^2-v_2^2\left(\frac{A_2}{A_1}\right)^2$$

And when you factor out the v_2^2 from the right side, you get:

$$2g\left[\left(\frac{P_1-P_2}{\gamma}\right)+(z_1-z_2)-h_L\right]=v_2^2\left(1-\left(\frac{A_2}{A_1}\right)^2\right)$$

Which leads to the equation for v_2 as follows:

$$\sqrt{\frac{2g\left[\left(\frac{P_1-P_2}{\gamma}\right)+(z_1-z_2)-h_L\right]}{\left(1-\left(\frac{A_2}{A_1}\right)^2\right)}}=v_2$$