

## Chapter 7

### General Energy Equation - Adding Power to the System



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## Review of Bernoulli

- Original Equation:

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

- No energy loss due to friction
- No mechanical devices

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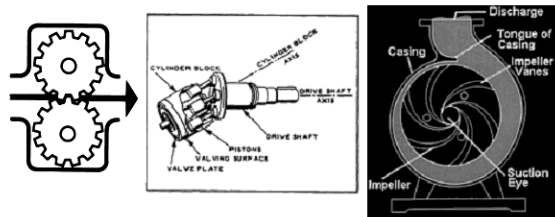
## Expanding on Bernoulli

- Energy losses and additions
  - Pumps
  - Fluid motors
  - Fluid friction
  - Valves and fittings
- All can change the energy state

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## Pumps

- Gear      Piston      Centrifugal



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## Fluid Motors

- Motors driven by fluid flow
- Hydraulic drives – Low speed, high torque: Earthmoving Equip.
- Waterwheels – Hydroelectric dams
- Hydraulically powered tools running off of back-hoe or tractor PTO (hydraulic Power-Take-Off)

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## Fluid Friction

- Loss of energy in a system due to friction along pipe walls
- Friction of fluid shearing against itself
- This energy is usually lost as thermal energy (heat).

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## Valves and Fittings

- Change in direction of flow
  - Elbows, Tees
- Energy loss going through valves
  - Check valves are spring loaded
  - High friction due to partially closed valve
- Another loss as thermal energy (heat).

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## Variables added to Bernoulli

- $h_A$  = Energy added (pump)
  - Sometimes called pump head
- $h_R$  = Energy removed (fluid motor)
  - Energy will be in feet or N-m/N
- $h_L$  = Energy losses due to friction
  - Included minor losses due to valves and fittings

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## General Energy Equation

- $E_1 + h_A - h_R - h_L = E_2$
- Where:
 
$$E = \frac{p}{\gamma} + z + \frac{v^2}{2g}$$
- Therefore:
- $\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_A - h_R - h_L = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$

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## Page 197 - Fig. 7.6

- Energy variables are on left side
  - “Act” on fluid based on original conditions
- Addition and subtraction of energy is already factored in equation.
  - Pump energy is added
  - Motor and frictional energy is subtracted

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## Different Example Using Fig. 7.7

- Water exits 3 inch diameter nozzle at 400 gpm. Calculate energy loss due to friction in pipe and valves.

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_A - h_R - h_L = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

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## Rewrite Energy Equation

$$z_1 - h_L = z_2 + \frac{v_2^2}{2g}$$

- Solve for energy losses  $h_L$

$$h_L = (z_1 - z_2) - \frac{v_2^2}{2g}$$

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### Solution

$$(z_1 - z_2) = 0 - (-25) = +25 \text{ ft}$$

$$v_2 = \frac{\dot{Q}}{A_2} = \frac{400 \text{ gal}}{\text{min}} \times \frac{231 \text{ in}^3}{1 \text{ gal}} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1 \text{ ft}^3}{1728 \text{ in}^3} \times \frac{1}{0.0491 \text{ ft}^2}$$

$$\frac{v_2^2}{2g} = \left( \frac{18.1 \text{ ft}}{\text{sec}} \right)^2 \times \left( \frac{1}{2} \right) \times \left( \frac{\text{s}^2}{32.2 \text{ ft}} \right) = 5.1 \text{ ft}$$

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### Final Solution

$$h_L = (z_1 - z_2) - \frac{v_2^2}{2g}$$

$$h_L = 25 \text{ ft} - 5.1 \text{ ft} = 19.9 \text{ ft}$$

- Book problem had less loss (like smoother, newer pipes), and greater flow rate (538 gpm vs. 400 gpm).

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### Different Example using Fig 7.8

- Assume the pump adds 55 N-m/N of energy at a flow rate of 0.025 m<sup>3</sup>/s, and losses due to piping and check valve is 3.0 N-m/N. What is the pressure at B?

$$\frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g} + h_A - h_R - h_L = \frac{P_B}{\gamma} + z_B + \frac{v_B^2}{2g}$$

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### Rewrite Energy Equation

$$\frac{P_B}{\gamma} = \frac{P_A}{\gamma} + (z_A - z_B) + \frac{(v_A^2 - v_B^2)}{2g} + h_A - h_L$$

Where:

$$P_A = -28 \text{ kPa}$$

$$g = 9.81 \text{ m/s}^2$$

$$\text{Sp Gr} = 0.86$$

$$h_A = 55 \text{ N-m/N}$$

$$z_A - z_B = -1.0 \text{ m}$$

$$h_L = 3.0 \text{ N-m/N}$$

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### Solve Continuity Equation

- Q(flow rate) = Av = A<sub>A</sub>v<sub>A</sub> = A<sub>B</sub>v<sub>B</sub>
- And v = Q(flow rate)/Area, then:

$$v_A = \frac{0.025 \text{ m}^3}{\text{s}} \times \frac{1}{4.768 \times 10^{-3} \text{ m}^2} = 5.24 \text{ m/s}$$

$$v_B = \frac{0.025 \text{ m}^3}{\text{s}} \times \frac{1}{2.168 \times 10^{-3} \text{ m}^2} = 11.5 \text{ m/s}$$

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### Solve Energy Equation

$$\frac{P_B}{0.86\gamma} = \frac{-28 \text{ kPa}}{0.86\gamma} + (-1.0 \text{ m}) + \frac{(5.24^2 - 11.5^2) \text{ m}^2/\text{s}^2}{2(9.81 \text{ m/s}^2)} + 55 \text{ N-m/N} - 3.0 \text{ N-m/N}$$

$$\frac{P_B}{0.86\gamma} = \frac{-28 \text{ kPa}}{0.86\gamma} + 45.66 \text{ N} \cdot \text{m/N}$$

$$P_B = 357 \text{ kN/m}^2 = 357 \text{ kPa}$$

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## Compare with Example 7.2

- Higher flow rate increases velocity
- Higher velocity increases friction
- More energy “lost” to friction
- Less pressure at Position B
- Common problem in piping: The more you push through, the greater the pressure drop at the other end.

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## Power Required by Pumps

- Power = rate of doing work
- Power added =  $h_A W$  (weight flow rate)
- Weight flow rate =  $\gamma Q$  (Vol/s)
- Therefore,  $P_A = h_A \gamma Q$
- Power in SI = Watts (= N-m/s)

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## Power in US Units

- 1 hp = 550 ft-lb/s
- One horse can pull 55 lbs up a mine shaft at 10 ft per second.
- A champion cyclist can develop about 1/3 horsepower.
- 1 kW = 1.34 hp (about 1-1/3 hp)

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## Mechanical Efficiency in Pumps

- Efficiency = output/input
- **ALWAYS** less than 100%
- Power delivered to the fluid divided by power put into the pump equals the pump efficiency.
- Mechanical Eff =  $e_M = P_A/P_I$

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## Mechanical Efficiency in Fluid Motors

- $P_R = h_R W = h_R \gamma Q$
- Where  $P_R$  is the power delivered to the fluid motor (energy removed from the fluid)
- Motor efficiency =  $e_M = P_O/P_R$
- Where  $P_O$  = mechanical power output of the motor.
- Again, remember efficiency < 100%

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## Homework – Chapter 7

- Problems, Chapter 7:
- 16, 22, 31, 41
- Solve 16 & 22 by spreadsheet
- Initial reading of Chapter 8

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