Kinetic Energy of Flowing Fluid

- Def: Energy due to motion. Units of ft – lbf or J (i. e., N – m)

$$KE = \frac{mv_{ave}^2}{2g_c}$$

- Per unit mass. Units of (ft – lbf) / lbm fluid equals approx ft fluid or (N – m) / kg fluid

$$KE = \frac{v_{ave}^2}{2g_c}$$

Kinetic Energy per Unit Weight

- Substitute m =(weight w) (g / g_c) to get KE per unit weight:
- Units of (ft – lbf) / lbf fluid equals ft fluid or (N – m) / N fluid equals m fluid

$$KE = \frac{v_{ave}^2}{2g}$$

Potential Energy of Flowing Fluid

- Def: Gravitational pull on object. Same units as KE

  PE = (weight w)(height z)

- Substitute weight w = m (g / g_c) to get per unit mass.

$$PE = \frac{zg}{g_c}$$

Potential Energy per Unit Weight

- PE = z
- Same units as KE per unit weight

Flow Work to Get Fluid in or out of Chosen section of pipe

- Work = (Force) (Distance)
- But Force =(Pressure) (Flow Area)
- and (Flow Area) (Distance traveled) = Volume of fluid
- Therefore Work = PV

Flow Work per Unit Mass

- (P)(specific mass volume) = $\frac{P}{\rho}$
- Same units as KE per unit mass

Flow Energy per Unit Weight

- (Pressure) (Specific weight volume) = $\dfrac{P}{\gamma}$
- Same units as KE per unit weight

Example Flow Energy Calculation

- Pump discharges fluid at P_1 = 150 psig in pipe. Pipe drops fluid into tank at P_2 = 0 gage

$$\frac{\text{Flow Energy}}{\text{unit weight}} = \frac{P_2 - P_1}{\gamma} = \frac{(150-0)144\frac{\text{in}^2}{\text{ft}^2}}{62.4\frac{\text{lb}_f}{\text{ft}^2}}$$

- = 346 ft fluid.

1st Law for Steady – State, Open System

$$Q_{in} - Q_{out} + W_{in} - W_{out} = \Delta U + \Delta KE + \Delta PE + \Delta PV$$

- Assume negligible Q, W, and single, homogeneous fluid
- Assume no fluid friction
- Therefore, $\Delta U = 0$

End Up With Bernoulli Equation

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

- This is form per unit weight
- Each term has units of ft or m fluid

Terms in Bernoulli Equation

- Def: Pressure head = 1st term
- Def: Elevation head= 2nd term
- Def: Velocity head = 3rd term
- **Rule of Thumb: ΔPressure Head greater than Δ Elevation Head greater than Δ Velocity Head**

Alternative form of Bernoulli Equation

$$\frac{P_1}{\rho} + Z_1\frac{g}{g_c} + \frac{V_1^2}{2g_c} = \frac{P_2}{\rho} + Z_2\frac{g}{g_c} + \frac{V_2^2}{2g_c}$$

- This is form per unit mass
- Each term given same name as that of unit weight form
- Each term is ft fluid head or (N-m) /kg fluid

Sample Problem – Flow out opening on Side of Tank (Fig 6.09 in text)

- Surface of Liquid(Point 1): P1 = 0 gage, velocity down v1 is negligible if d1 much greater than d2, elevation = z1 above opening
- Outside opening (Point2) P2 = 0 gage, z2 = 0

Sample Problem 1 Concluded

- Plug the unknowns into Bernoulli equation per unit weight and solve for v_2

$$v_2 = \sqrt{2gz_1}$$

Sample Problem 2 – Fig 6.11 in text

- How high does jet shoot upward from opening?
- At surface of liquid (Point 1): P1 = 0 gage, v1 down is negligible, elevation z1 is h.
- At top of jet (Point2): P2 = 0 gage, v2 = 0.

Sample Problem 2 – Concluded

- Plug knowns into Bernoulli per unit weight and solve for h2.
- Obtain h2 = h1

Sample Problem 3 – Tank in example 2 is pressurized

- How much pressure P1 required to shoot jet upwards 40 ft?
- At surface of liquid: P1 = ? gage, v1 downward is negligible., h1 is 6 ft
- At top of jet: Again v2 = 0, p2 = 0 gage. But now h2 = 40 ft.

Sample Problem 3 – Concluded

- Plug knowns into Bernoulli per unit weight and solve for P1
- Obtain $\dfrac{P_1}{\gamma} = 40 - 6 = 34\text{ft}$
- Or $P_1 = 34\gamma = 2122\ lbf/ft^2$ gage

Sample Problem 4 – Flow between pipes of different sizes

- Two horizontal pipes in series. Pipe 1 is 1inch id, velocity v1 is 10 ft/s, Pressure is P1. Pipe 2 is larger (2 inch id) Velocity 2 is ? Pressure P2 is ?
- Continuity says
- $$v_2 = v_1 \frac{A_1}{A_2} = v_1 \left(\frac{D_1}{D_2}\right)^2 = 2.5 \text{ ft/s}$$

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Sample Problem 4 Concluded

- Bernoulli says

$$\frac{P_2 - P_1}{\gamma} = (z_2 - z_1) + (\frac{v_1^2 - v_2^2}{2g})$$

- Or

$$P_2 - P_1 = (1.45 \text{ft head})(\gamma / 144) = 0.6 \text{ psi}$$

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Sample Problem 5 – Venturi Meter on P.169 of text

- Venturi meter is device to measure flow rate
- It consists of differential manometer between two pipes of known inner diameters. The manometer measures the the pressure drop from the bigger to the smaller pipe.

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Sample Problem 5 Continued

- At Point A of bigger pipe:D_A = 0.3m.
- At Point B of smaller pipe: D_B = 0.2 m
- To get $P_A - P_{B:}$ Let Point 1 = meniscus on left side of manometer & Point 2 = same elevation on right side of manometer

$$P_1 = P_A + (1.18 + y)\gamma$$

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Sample Problem 5 Continued

$$P_2 = P_1 = P_B + (0.46 + y)\gamma + (1.18)(1.25sg)\lambda$$

- Or $P_A - P_b = 0.755\ \gamma$
- From Continuity:
- $$v_B = v_A \left(\frac{D_A}{D_B}\right)^2 = 2.25 v_A$$

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Sample Problem 5 Concluded

- Substituting these two results into Bernoulli:

$$\frac{P_A - P_B}{\gamma} + (z_A - z_B) = \frac{(2.25^2 - 1)v_A^2}{2g}$$

$$or\ 0.755\text{m} - 0.46\text{m} = \frac{4.064 v_A^2}{2(9.81)}$$

$$or\ v_A = 1.19 \text{m/s}$$

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Homework for Next Class

- 6.69PartA(want m^3 /s),
- 6.69 part B (want kPa gage for A & B). A is inside pipe at height of the liquid surface)