

## Chapter 5

### Buoyancy

Miami University

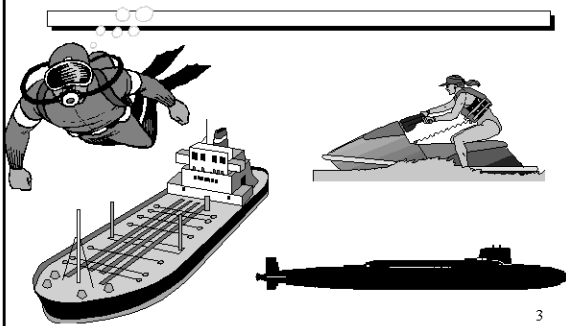
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## Definition

- Any body in a fluid, whether floating or submerged, is buoyed up by a force equal to the weight of the fluid that has been displaced.

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## All objects react to buoyancy



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## Force due to buoyancy

- $F_b = \gamma_f \cdot V_d$
- Buoyancy is a force equal to the fluid's specific weight times the volume of fluid displaced.
- Archimedes' Principle

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## Example of buoyancy force

- A 6 inch cube of steel is submerged underwater. How much force does the water exert on the cube?  $F_b = \gamma_f \cdot V_d$

$$F_b = \frac{62.4 \text{ lbf}}{\text{ft}^3} \times (0.5 \text{ ft})^3 = 7.8 \text{ lbf}$$

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## Sink or swim?

- If the object weight ( $w$ ) is greater than the buoyancy force ( $F_b$ ), then the object will sink.
- If the object weight is less than the buoyancy force, then the object will float.

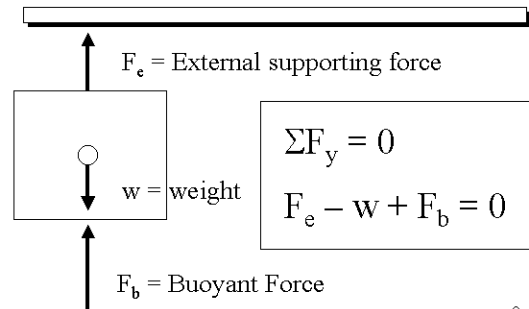
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## Ocean liners and air mattresses

- Same buoyancy principle keeps both objects afloat. Huge ships displace more water weight than their own mass.
- An air raft allows you to float due to the large area of water being displaced. A 3 ft x 6 ft x 2 inch (into water) raft displaces 187 lbs of water.

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## Free Body Diagram of Steel Cube



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## External Force to suspend cube

- $F_e - w + F_b = 0$
- $F_e = w - F_b$

$$F_e = \frac{0.283 \text{ lbf}}{\text{in}^3} \times (6 \text{ in})^3 - F_b$$

$$F_e = 61.1 \text{ lbf} - 7.8 \text{ lbf} = 53.3 \text{ lbf}$$

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## Determining Volume of Odd Shapes

- Use the principle of buoyancy to determine volume.
- Determine the weight of the object dry and submerged, then use the FBD equation to solve for volume.

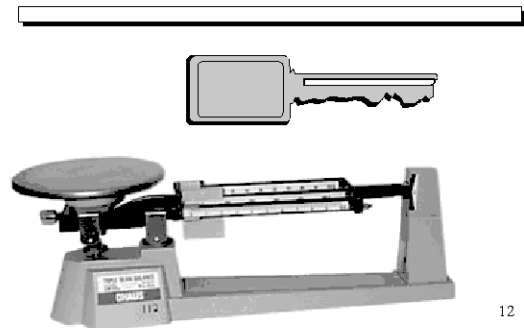
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## Volume and Density Example

- Determine the volume and density of a metal car ignition key.
- Weigh the key on a balance.
- Weigh the key suspended in water
- Solve ( $F_e - w + F_b = 0$ ) for  $F_b$
- Then  $V = F_b / \gamma_{\text{water}}$

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## Demonstration



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## Floating Objects

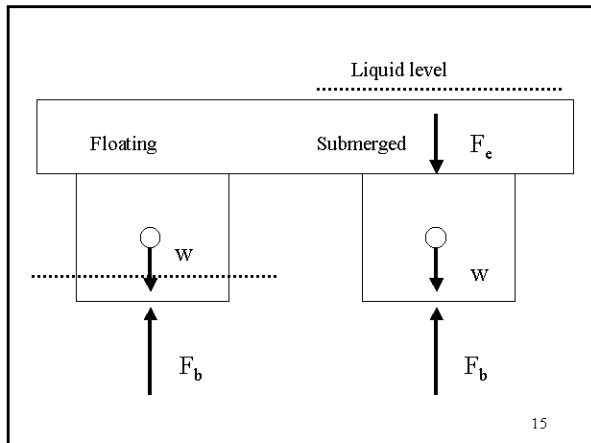
- FBD equation can determine how much of the object is out of the liquid
- FBD equation can determine how much force is required to fully submerge the object

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## Floating Object Example

- A 10 inch wood cube floats in water with 2 inches below the surface.
- Determine the force required to submerge the block in gasoline (sp.gr.= 0.68)

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## Floating Block

$$w = F_b = \gamma_f \cdot V_d$$

$$F_b = \frac{62.4 \text{ lbf}}{\text{ft}^3} \times \frac{(10 \times 10 \times 2) \text{ in}^3}{1728 \text{ in}^3 / \text{ft}^3}$$

$$F_b = 7.2 \text{ lbf}$$

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## Submerged Block

$$F_b - F_e - w = 0$$

$$F_e = F_b - w = (\gamma_f \cdot V_d) - 7.2 \text{ lbf}$$

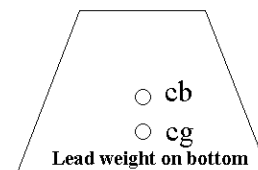
$$F_e = \left[ 0.68 \times \frac{62.4 \text{ lbf}}{\text{ft}^3} \times \frac{(10 \text{ in})^3}{1728 \text{ in}^3 / \text{ft}^3} \right] - 7.2 \text{ lbf}$$

$$F_e = 17.4 \text{ lbf} \text{ downward force}$$

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## Stability of Submerged Objects

- The center of gravity must be **below** the center of buoyancy to remain stable while submerged.



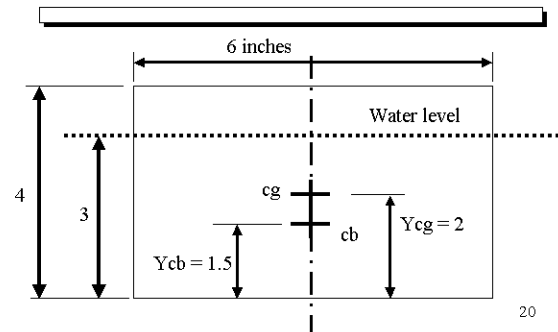
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## Center of Buoyancy

- Located at the centroid of the displaced volume of liquid
  - For submerged objects, typically at the centroid of the object.
  - For floating objects, center of buoyancy is the centroid of the displaced liquid volume only.

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## Locating cg and cb



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## Stability of Floating Objects

- A floating object is stable if its center of gravity is **below** its metacenter
- The metacenter is a point on the vertical centerline of the object which is always directly above the buoyant force when the object is slightly rotated.

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## Determining the metacenter

- Distance from the buoyancy center to the metacenter = MB
- $MB = I/V_d$ , where
- $I$  = Least Moment of Inertia of the object at the liquid surface plane
- $V_d$  = Volume of displaced fluid

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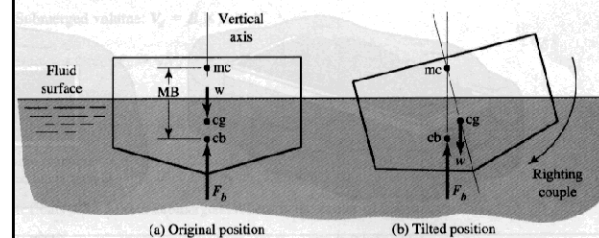
## Determining Stability

- $y_{mc} = y_{cb} + MB$
- If  $y_{mc} > y_{cg}$ , the body is stable.
- If  $y_{mc} < y_{cg}$ , the body is unstable.

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## 5.5 Stability of Floating Bodies

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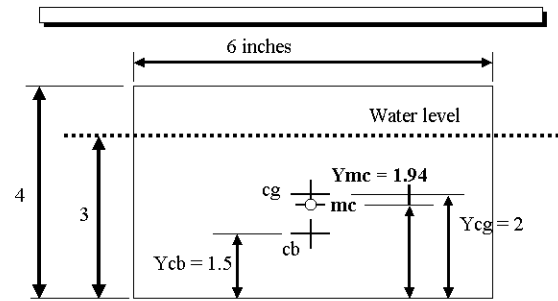
### Determining Metacenter

- Assume the block in Slide 20 has a 4 x 6 inch base.
- $MB = I/V_d = (bh^3/12)/(\text{displaced } V)$

$$MB = \frac{(6 \times 4^3)/12}{(4 \times 6 \times 3)} = 0.44 \text{ in}$$

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$$Y_{mc} = Y_{cb} + MB$$



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### Stability in Vessels

- Metacentric height ( $Y_{mc} - Y_{cg}$ ) is 1.5 ft (min.) for small vessels and 3.5 ft (min.) for large ships, but an excessive MG will cause too much rocking.

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### Homework – Chapter 5

- Problems, page 133:
- 2M, 6M, 21M, 35E, 41E, & 46E
- Initial reading of Chapter 6
- 2<sup>nd</sup>, Detailed Reading of Chap 5

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