

Forces Due to Static Fluids

- Def: Force developed due to pressure over a set area by a fluid not in motion
- The force varies when the set area is at a varying liquid depth
- We want to know the resultant force and the position of its horizontal plane of action(i.e., center of pressure)

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Examples

- Force against the face of a dam
- Force on the bottom of a liquid storage tank
- Force on the sides of a liquid storage tank
- Force on pneumatic or hydraulic cylinder

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Forces Due to Gases (under Pressure)

- Gases have low specific weight
- So Variation of pressure with depth can be neglected for typical situations
- Force = (Gas Pressure) (Area exerted)
- Areas typically is a circle

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Examples of Gas Force on Cylinder

- At board

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Forces Due to Liquids(1)

- Liquids have high specific weight, so variation of pressure with depth is significant
- Force = (Constant or Average Pressure) (Wetted Area)

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Forces Due to Liquids(2)

- Wetted area may depend on whether surface is horizontal, vertical, inclined, or partially wet
- Plane of action depends on the number of layers and is different (closer to bottom) than the depth having the average pressure

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Forces Due to Liquids(3)

- Usually we want to know the additional force from the liquid and don't include the gas pressure on the surface of the top liquid layer
- Later we will show how to include this gas surface pressure as well

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Horizontal Flat Surface under Liquids(1)

- Normally have only one liquid layer in a tank
- Pressure = Constant(not an ave.) = (Depth) λ
- With multiple layers, add (Depth) λ for each layer
- Force = (Pressure) (Wetted Area). 8

Horizontal Flat Surfaces Under Liquids (2)

- Note pressure independent of the shape of the vessel holding the liquid.
- Shape does affect the bottom area, however.

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Examples with Multiple Liquid Layers

- At Board

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Side Walls (like of tank, dam) (1)

- Pressure increases with depth $h = \Delta p = \gamma \Delta h$
- Force = P_{ave} x Wetted Area
- but $P_{ave} = \gamma (h / 2)$ where $h =$ depth
- so Force = $\gamma (h / 2)$ (Wetted Area)

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Side Walls (2)

- Force always acts perpendicular to the wall at the center of pressure
- Assuming single liquid, center of pressure is 1/3 liquid depth from the bottom. This is lower than 1/2 depth corresponding to ave Pressure
- Why? Greater portion of force acts near the bottom than the top.

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Side Wall Example

- 20 ft diam, 40 ft high cylindrical tank that is $\frac{1}{4}$ full with water and open to air. Force on side walls?
- $P_{ave} = (10/2 \text{ depth})(\gamma \text{ of } 62.4)$ at 5 ft from bottom
- Wetted side wall area = $(\pi)(20 \text{ diam})(10 \text{ depth})$
- Side Force = $(P_{ave})(\text{Area}) = 196,000 \text{ lbf}$. due to liquid
- Center of Pressure = $(10/3)$ ft from bottom

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Side Wall (Dam) Example

- Earth dam, 50 deg angle, 100 m across, 6 meters deep water. Force on dam?
- Wetted dam area = $(6/\sin 50)(100 \text{ across})$
- $P_{ave} = (6/2)(\gamma \text{ of } 9.81 \text{ kN/m}^3)$ at 3 m from bottom
- Force on Dam = $(P_{ave})(\text{Area}) = 23 \text{ MN}$
- Center of Pressure = $6/3 = 2 \text{ m}$ from bottom

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Submerged, Vertical Window Example

- On Newport aquarium, 2 ft diameter window, whose top edge is 6 ft below water surface. Force on Window?
- Wetted window area = $(\pi/4)(2 \text{ diam})^2 = \pi \text{ ft}^2$
- $P_{ave} = (6 + 1 \text{ ave depth})(\gamma \text{ of } 62.4 \text{ lbf/ft}^3)$ at 7 ft from surface
- Force on window = $(P_{ave})(\text{Area}) = 1372 \text{ lbf}$
- Center of Pressure = depth of $8 - (1/3)(2) = 7 \frac{1}{3}$ ft below surface

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Submerged, Inclined Area Example

- 12 ft diam cylindrical tank with conical bottom. Cone has side of 10 ft and adds 8 ft to the 14 ft depth in the cylinder above. Force on cone?
- Wetted area of cone = $(1/2)(\text{perimeter})(\text{side}) = 0.5(\pi \times 12 \text{ diam})(10 \text{ ft side}) = 188 \text{ ft}^2$
- $P_{ave} = (14 + 8/2 \text{ depth})(\gamma \text{ of } 62.4 \text{ lbf/ft}^3) = 1123 \text{ lbf/ft}^2$ at 18 ft below surface
- Force on cone = $(P_{ave})(\text{Area}) = 211,000 \text{ lbf}$

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Including the Pressure (P_s) on the Surface of the Liquid

- May be above or below atmospheric on surface in closed tank
- Easy way to include: Convert the gage pressure (+ or -) to equiv. depth of the liquid = P_s/γ
- Add this additional depth to that of the liquid layer(s) for calculating the pressure (or average pressure), the force, and center of pressure.

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Example of pressurized tank

- Previous 20 ft diam cylindrical tank, pressurized at 5 psig,
- Equiv depth = $(5 \text{ lbf/in}^2)(144 \text{ in}^2/\text{ft}^2)/(\gamma \text{ of } 62.4 \text{ lbf/ft}^3) = 11.5 \text{ ft}$
- P_{ave} on Sides = $(10 + 11.5)/2$ ave depth (γ of 62.4) at 10.75 ft from bottom
- Wetted Side Area (same as previous) = 628 ft²
- Side Force = $(P_{ave})(\text{Wetted Area}) = 421,000 \text{ lbf}$
- Center of pressure = $(10 + 11.5)/3 \text{ ft} = 7.2 \text{ ft}$ from bottom

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Homework for Next Class

- Problems, page 97: 4.3, 4.11, 4.17
- Initial Reading of Chap 5
- 2nd, Detailed Reading of Chap 4