Miami University and
University of Cincinnati.

Donald Ulrich, MS Chem.Eng.
Adjunct Instructor

USEFUL EQUATIONS FOR INCOMPRESSIBLE FLUID FLOW – Rev 19

Bernoulli equation Per unit Weight:

Key advantage of this form of Bernoulli Equation: Whether in US or metric units, each term has units of a height of fluid

Equation (1):

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_{ave1}^2}{2g} + \text{Work}_P - \text{Work}_T = \frac{P_2}{\gamma} + Z_2 + \frac{V_{ave2}^2}{2g} + \frac{\text{fluid friction losses}}{\text{unit weight}}$$

Each term including the Work has dimensions of ft - lb_f per lb_f = ft head of fluid or (newton - meters) / newton = joules / newton = meters head of fluid

Definitions: γ = weight density in lb_f / ft^3 or newtons / m^3

Z is the height in ft or m above some arbitrary plane

g = local gravity g is 32.2 ft / s^2 or 9.81 m / s^2 on earth. Varies less than 0.5% around the earth

g_c is a conversion factor. Value is 32.174 (ft - lb_m) / (lb_f - s^2) or 1 (kg - m) / (newton - s^2) or 1 (slug – ft) / (lb_f – s^2)

V_{ave} = average fluid velocity in ft / s or m / s

P = Pressure P in lb_f / ft^2 or pascals which is newtons / m^2

Work = Work per unit weight = ft head or meters head = is the Work in W_P (pump work delivered to the fluid) or Work out W_T (work done by fluid on turbine) per unit weight of fluid

Converting Head Fluid to Pressure = (Head) γ where γ is the weight density of the fluid in lb_f / ft^3 or newtons / m^3

Bernoulli Equation Per unit Mass:

Key disadvantage of this form of Bernoulli Equation: in metric, units of each term are NOT a height of fluid.

Equation (2):

$$\frac{P_1}{\rho} + Z_1 \frac{g}{g_c} + \frac{V_{ave1}^2}{2g_c} + \text{Work}_P - \text{Work}_T = \frac{P_2}{\rho} + Z_2 \frac{g}{g_c} + \frac{V_{ave2}^2}{2g_c} + \frac{\text{fluid friction losses}}{\text{unit mass}}$$

Each term including the work has dimension (ft - lb_f) per lb_m = feet head of fluid or (newton - meter) / kg = joules / kg which is not meters head of fluid. Alternative US unit is (ft – lb_f) / slug mass

Definitions: ρ = mass density in lb_m / ft^3 or kg / m^3. Alternative US units is slugs / ft^3

Work_P and Work_T are work per unit mass of fluid

Other definitions are same as above under Bernoulli equation per unit weight

Converting Head Fluid (or (newton – meters) / kg) to Pressure = (Head) ρ where ρ is the mass density of the fluid in lb_m / ft^3 or slugs / ft^3 or newtons / m^3

USEFUL EQUATIONS FOR INCOMPRESSIBLE FLUID FLOW – Rev 19

Another Variation of Bernoulli Equation in Some Texts:

We divide each term of Equation 2 by g/ g_c and in effect covert equation per unit mass to equation per unit weight

$$\frac{P_1}{\rho}\left(\frac{g_c}{g}\right)+Z_1+\frac{V_{ave1}^{\ 2}}{2g}+\text{Work}_P-\text{Work}_T=\frac{P_2}{\rho}\left(\frac{g_c}{g}\right)+Z_2+\frac{V_{ave2}^2}{2g}+\frac{\text{fluid friction losses}}{\text{unit weight}}$$

Each term including the Work has dimensions of ft - lb_f per lb_f = ft head of fluid or (newton - meters) / newton = joules / newton = meters head of fluid

More on Pump Work (in to fluid) or Turbine Work (out from fluid):

$\eta W = W_P$ where W is the brake work of the pump, η is the fraction of brake work delivered to the fluid, & W_P equals work delivered to the fluid

$W = \eta W_T$ where W is the turbine work recovered from the fluid work W_T on the turbine & η is the fraction of fluid work recovered from the turbine

Power $\dot{W}$ delivered to the fluid or turbine:

With Work per unit weight, Power = $\dot{W} = W_{P \text{ or } T}\,\dot{V}\,\gamma = W_{P \text{ or } T}\,\dot{w}$ = in (ft - lb_f) / s or joules / s = watts

With Work per unit mass, Power = $\dot{W} = W_{P \text{ or } T}\,\dot{V}\,\rho = W_{P \text{ or } T}\,\dot{m}$ in (ft - lb_f) / s or joules / s = watts

Definitions: Work per unit weight in ft head or meters head. Work per unit mass in (ft – lb_f) / lb_m and (ft – lb_f)/ slug or (newton – meters) / kg = joules / kg

$\dot{V}$ = Volumetric flow rate = m^3 / s or ft^3 / s
γ = Specific weight = Weight density = lb_f / ft^3 or newtons / m^3
ρ = mass density = lb_m / ft^3 or slugs / ft_3 or kg./ m^3
$\dot{w}$ = weight flow rate = lb_f / s or N / s
$\dot{m}$ = mass flow rate = lb_m / s or kg / s

Reynolds Number

Equation (3) for Reynolds Number, a Dimensionless Number $= N_{Re} = \dfrac{\rho V D}{\mu} = \dfrac{VD}{\upsilon}$

USEFUL EQUATIONS FOR INCOMPRESSIBLE FLUID FLOW – Rev 19

Equation (4) for Reynolds number in circular pipe (only) = $N_{Re} = \frac{4\rho\dot{V}}{\pi D\mu} = \frac{4\dot{V}}{\pi D\upsilon}$

Definitions: ρ = mass density –= lb_m / ft^3 or kg / m^3
V = average fluid velocity in ft / s or m / s
D = inner pipe diameter or equivalent diameter given by Equation 5 below in ft or m
μ = dynamic viscosity = lb_m / (ft – s) or kg / (m – s) which equals Pa – s and (N / m^2) - s
ν = kinematic viscosity in ft^2 / s or m^2 / s = μ / ρ = dynamic viscosity divided by mass density
$\dot{V}$ = Volumetric flow rate = ft^3 /s or m^3 / s

Equation (5) for Equivalent Diameter of Non Circular Duct

Use $D = \frac{4(\text{Cross Sectional Area for flow})}{\text{Wetted Perimeter}}$ in Reynolds number

Examples: for a square duct of side s, d = s
For annulus, $D = D_2 - D_1$ where D_2 is the inner diameter of the outer pipe and D_1 is the outer diameter of the inner pipe.
For rectangular duct of height a and width b, $D = \frac{2ab}{a+b}$

Calculating Reynolds Number in Typical American units to get dimensionless number

Mass density

$$N_{Re} = \frac{(lb_m / ft^3)(ft)(ft/s)}{lb_m/(ft - s)} = \frac{(lb_m / ft^3)(ft)(ft/s)}{(\frac{lb_f}{ft^2} - s)(\frac{32.2 lb_m/(ft - s)}{(\frac{lb_f}{ft^2}) - s})}$$

Viscosity

Calculating Reynolds Number in Alternative American Units to get dimensionless number

Mass density

$$N_{Re} = \frac{(slugs/ft^3)(ft)(ft/s)}{(lb_f / ft^2) - s} = \frac{((lb_f - s^2)/ft^4)(ft)(ft/s)}{(lb_f / ft^2) - s}$$

Viscosity

Since slugs / ft3 mass density also equals (lb_f - s^2) / ft^4

Calculating Reynolds Number in SI units to get dimensionless number

Mass Density

USEFUL EQUATIONS FOR INCOMPRESSIBLE FLUID FLOW – Rev 19

$$N_{Re} = \frac{(kg/m^3)(m/s)(m)}{Pa - s} = \frac{(kg/m^3)(m/s)(m)}{kg/(m - s)}$$

Viscosity

Since Pa-s viscosity is equivalent to kg /(m - s)

Continuity equations for Incompressible Fluid Through a Single Pipe (Subscript 1) or Two Pipes in Series (subscripts 1 & 2): Equations tells us the mass flow rate, weight flow rate, and volumetric flow $\dot{V}$ do not change throughout a single pipe or pipes in series.

Equations (6) for flow rates through any shape of a single pipe or pipes in series:

$\dot{V}_1 = \dot{V}_2$ **and** $\dot{m}_1 = \dot{m}_2$ **and** $\dot{w}_1 = \dot{w}_2$

where $\dot{m} = \rho \dot{V}$ **and** $\dot{w} = \gamma \dot{V}$

Definitions: $\dot{V}$ = Volumetric flow rate in ft^3 / s or m^3 / s
$\dot{m}$ = mass flow rate = lb_m / s or kg / s
$\dot{w}$ = weight flow rate = lb_f / s or N / s
ρ = mass density in lb_m / ft^3 or kg / m^3
γ = weight density in lb_f / ft^3 or newtons / m^3

Equation (7) for average velocity V_{ave} through any shape of a single pipe or pipes in series:

$\dot{V}_1 = \dot{V}_2$ = Volumetric flow rate = same throughout single pipe or each pipe in series = Cross Sectional $Area_1$ x Average $Velocity_1$ = Cross Sectional $Area_2$ x Average $Velocity_2$

$= A_1 \, v_{ave1} = A_2 \, v_{ave2}$

Definitions: $\dot{V}$ = Volumetric flow rate in ft^3 / s or m^3 / s
A = Cross Sectional area in ft^3 or m^3
V_{ave} = Average fluid velocity in ft / s or m / s

Equation (8) for average velocity V_{ave} through circular pipes in series

USEFUL EQUATIONS FOR INCOMPRESSIBLE FLUID FLOW – Rev 19

$\dot{V}_1 = \dot{V}_2 = \frac{\pi}{4} D_1^2 \ V_{ave1} = \frac{\pi}{4} D_2^2 V_{ave2}$ = Volumetric flow rate = same throughout a single pipe or each pipe in series

Definitions: . $\dot{V}$ = Volumetric flow rate in ft^3 / s or m^3 / s
D -= Circular pipe inner diameter in ft or m
V_{ave} = Average fluid velocity in ft / s or m / s

Continuity equation for Compressible Fluid through any shape of a Single pipe (Subscript 1) or Two Pipes in Series (Subscripts 1 and 2)

Equation (9) for Mass Flow Rates $\dot{m}$ in single pipe or pipes in series = same throughout a single pipe or each pipe in series:

$$\dot{m}_1 = \rho_1 \dot{V}_1 = \rho_1 V_{ave1} A_1 = \dot{m}_2 = \rho_2 \dot{V}_2 = \rho_2 V_{ave2} A_2 \quad \text{in units of } lb_m / s \text{ or } kg / s$$

Definitions: $\dot{m}$ = mass flow rate = lb_m / s or kg / s
ρ = mass density in lb_m / ft^3 or kg / m^3
$\dot{V}$ = Volumetric flow rate in ft^3 / s or m^3 / s
A = cross sectional area of flow in ft or m
V_{ave} = average fluid velocity in ft /s or m / s

Equation (10) for Weight Flow Rates $\dot{w}$ in single pipe or pipes in series = same throughout a single pipe or each pipe in series:

$$\dot{w}_1 = \gamma_1 \dot{V}_1 = \gamma_1 V_{ave1} A_1 = \dot{w}_2 = \lambda_2 \dot{V}_2 = \gamma_2 V_{ave2} A_2 \quad \text{in units of } lb_f / s \text{ or } N / s$$

Definitions: $\dot{w}$ = weight flow rate = lb_f / s or N / s
γ = weight density in lb_f / ft^3 or newtons / m^3
$\dot{V}$ = Volumetric flow rate in ft^3 / s or m^3 / s
A = cross sectional area of flow in ft or m
V_{ave} = average fluid velocity in ft /s or m / s

USEFUL EQUATIONS FOR INCOMPRESSIBLE FLUID FLOW – Rev 19

Equation for Frictional Losses in Pipes or Duct

Equation (11) in terms of Average Velocity: $\frac{\text{losses}}{\text{unit mass}} = f\frac{L}{D}\frac{V_{ave}^{\;2}}{2g_c}$ and

$$\frac{\text{losses}}{\text{unit weight}} = f\frac{L}{D}\frac{V_{ave}^{\;2}}{2g}$$

Definitions: Losses / unit mass has units of ft - lb_f / lb_m or J / kg
Losses per unit weight has units of ft – lb_f / lb_f = ft head of fluid or (N – m) / N = m head of fluid
f = is the Darcy friction factor. NOTE the Darcy friction factor = 4 times the Fanning friction factor
D = inner diameter in ft or m for a circular pipe. For non-circular pipe or duct, use equivalent D given by Equation (5) previously. Note: **Equation 5 strictly valid ONLY for the frictional losses with turbulent flow.**
L = Length in ft or m
V_{ave} = average fluid velocity V in ft. per s or m/ s
g_c = conversion factor of 32.174 (ft - lb_m) / (lb_f - s^2) or 1 (kg – meter)/ (newton - s^2)
g= local acceleration of gravity = about 9.8 m / s^2 or 32.2 ft / s^2

Equation for Circular Pipes (Only) In Terms of Volumetric Flow Rate $\dot{V}$

If we solve for V_{ave}, the average velocity of flow, in terms of the diameter of the a circular pipe and the volumetric flow $\dot{V}$ (ft^3 / s or m^3 / s), and then plug this into the general equation for frictional pipe losses above, we get a very useful expression:

Equation (12) $\frac{\text{frictional losses}}{\text{unit mass}} = 8f\frac{L}{D^5}\frac{\dot{V}^2}{\pi^2 g_c}$ units of (ft-lb_f)/ lb_m or J /kg

and $\frac{\text{frictional losses}}{\text{unit weight}} = 8f\frac{L}{D^5}\frac{\dot{V}^2}{\pi^2 g}$ units of (ft - lb_f)/ lb_f = ft head or m head

Definitions: f = Darcy friction factor = 4 times the Fanning friction factor
$\dot{V}$ = volumetric flow rate in ft^3 / s or m^3 / s
D -= circular pipe inner diameter in ft or m
g_c = conversion factor of 32.174 (ft - lb_m)/ (lb_f - s^2) or 1 (kg – meter) / (newton - s^2)
g = local acceleration of gravity = about 32.2 ft / s^2 or 9.8 m / s^2

Note the losses are proportional to the square of the volumetric flow rate $\dot{V}$ and inversely proportional to the pipe inner diameter raised to the 5th power!

USEFUL EQUATIONS FOR INCOMPRESSIBLE FLUID FLOW – Rev 19

Equations for Friction Factor f

Notes: Darcy and Fanning friction factor depends on Reynolds number for laminar flow & Reynolds number & surface roughness for turbulent flow as follows:

Equation (13) for Darcy friction factor with laminar flow $= \dfrac{64}{N_{\mathrm{Re}}}$

Where $N_{\mathrm{Re}} \le 2000$ defines laminar flow

Darcy friction factor for turbulent flow (Where $N_{\mathrm{Re}} \ge 2000$)

See Moody diagrams versus Reynolds number & the pipe's relative roughness (defined as ε/D (or the reciprocal of this) where ε is the roughness factor for the pipe in ft or m and D is the inner diameter (in ft. or m) of a circular pipe or equivalent diameter of another shape